Newton Mechanics and Basics of General Relativity in the Context of Cosmology Teaching:

classroom activities and their impact on learning & motivation

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Percorsi didattici interdisciplinari nell'OS FAM

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scnat
akademie der naturwissenschaften
académie des sciences naturelles
swiss academy of sciences
accademia di scienze naturali

SPS
swiss physical society

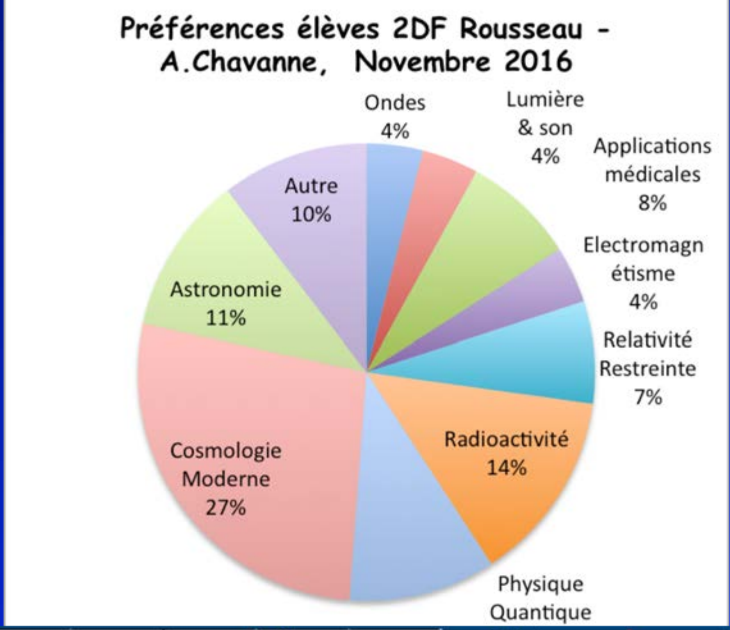
SwissMAP
The Mathematics of Physics
National Centre of Competence in Resear

Outline

- Motivation & objectives of the project
- Examples of activities
- Physics education research : main results
- Conclusions & open discussion

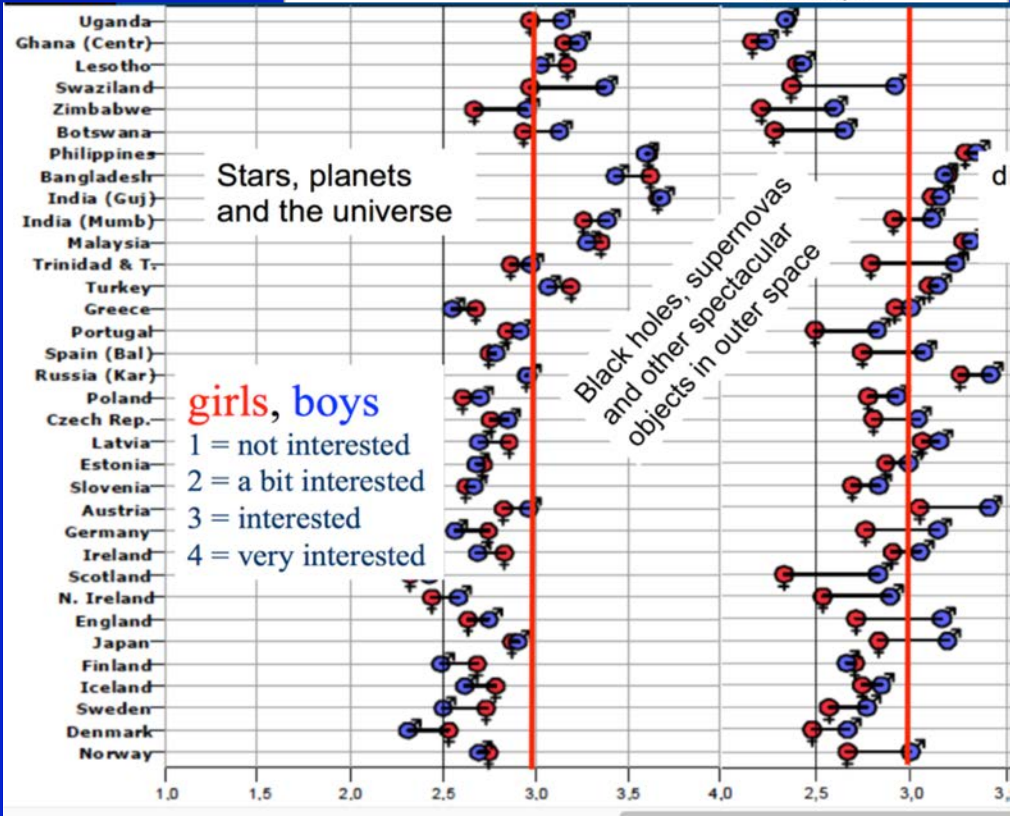
Motivation & Purposes

1. *__Boost__* the learners' level in maths and in traditionally taught physics (e.g. classical mechanics, waves, electromagnetism or thermodynamics) by attractive subjects: *not replace* but *complete* the "classical" curriculum



Research ROSE

(Relevance of Science Education)



- ➢ Similar results across many countries
- ➢ Averages for « ordinary » science subjects are about 2

Motivation & Purposes

2. *__Improve__* the links between high school and actual research

   Students who learn physics up to the 19th century have a distorted idea about the main issues of modern physics

3. *__Medium level__* of transposition

   Activities based on high school skills on maths and "classical" taught physics

   ⇒ neither for a wide public (zero equations), nor academic level

- Contents are based exclusively on the sec II sciences' study plan ("gymnase")
- Progressive level of difficulty

Teaching material

Course of 8 chapters

- Theory (book, 2018 & 2023 in FR; 2024 in IT; EN planned for march 2026)
- Exercises & activities with corrections

https://nccr-swissmap.ch/school-teachers-children/general-relativity

https://physalice.ch/cosmologie/

Table des matières

➔ Ideal for a physics optional course (2h/week), starting from senior high school

➔ Toolbox for many others teaching contexts (punctual activities)

➔ Complete intro for any interested learner with a good scientific background

Teaching material

Course of 8 chapters

- Theory (book, 2018 & 2023 in FR; 2024 in IT; EN planned for march 2026)
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Table des matières

❖ Course built and evolved over 10 years in parallel with the practice with classes in CH, FR, CA and BE (recognized by the international “Contemporary Physics Education Prize” in 2020)

❖ The content and structure of the course have been cross-evaluated and validated by academic experts (RG, cosmology and education) and as part of the publication of the book and scientific articles (see e.g. SPS n. 55 or SSPMP n. 137 our PRL)

Teaching material

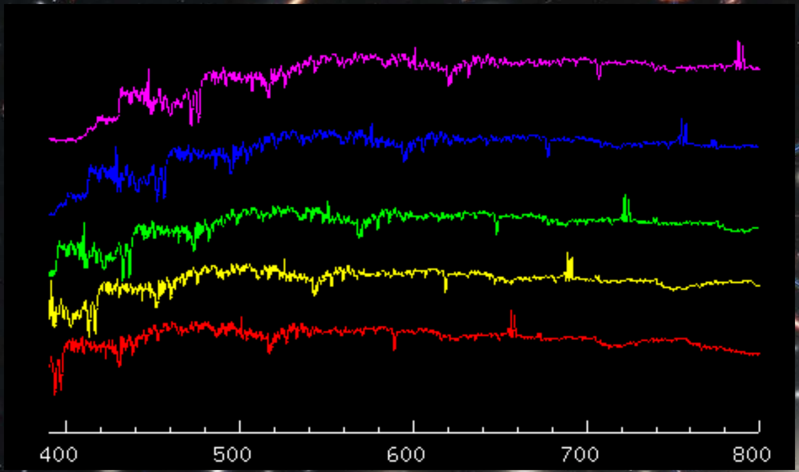
Different "thematic paths" can be explored through the chapters, following a chosen topic that can be developed by increasing the level of difficulty

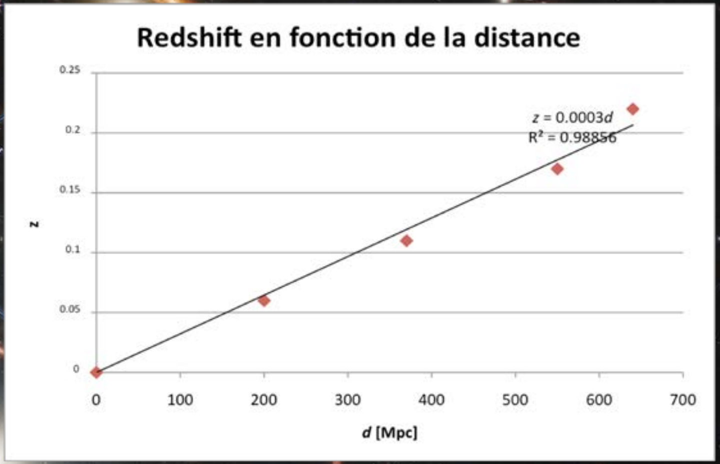
Examples :

1. Analogies & differences between gravitational and electromagnetic interactions
2. Black holes
3. Curvature & gravitational lensing
4. Expansion of the universe

Scales, expansion, redshift and distances

- Find Hubble-Lemaître law by comparing nearby galaxies spectra



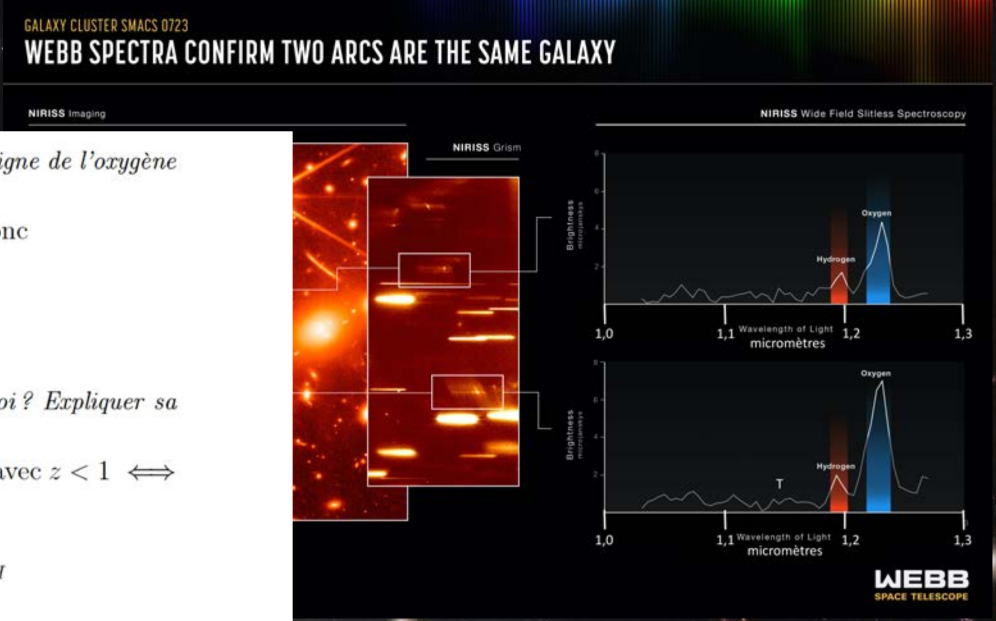


- Compare the rate of the expansion's speed at different scales

H_0 = 70 km/s /Mpc = … /km = … /Gpc

- Difference between
  - *Doppler effect* and *cosmological redshift*
  - *Hubble radius* ($z < 1$) and *horizon* ($z < \infty$)

Examples of activities



b) *En lisant dans le graphique la valeur de la longueur d'onde observée pour la ligne de l'oxygène OIII, calculer son redshift.*

On peut lire sur le graphique $\lambda_0 = 1,2\,\mu$m et on sait que $\lambda = 0,486\,\mu$m donc

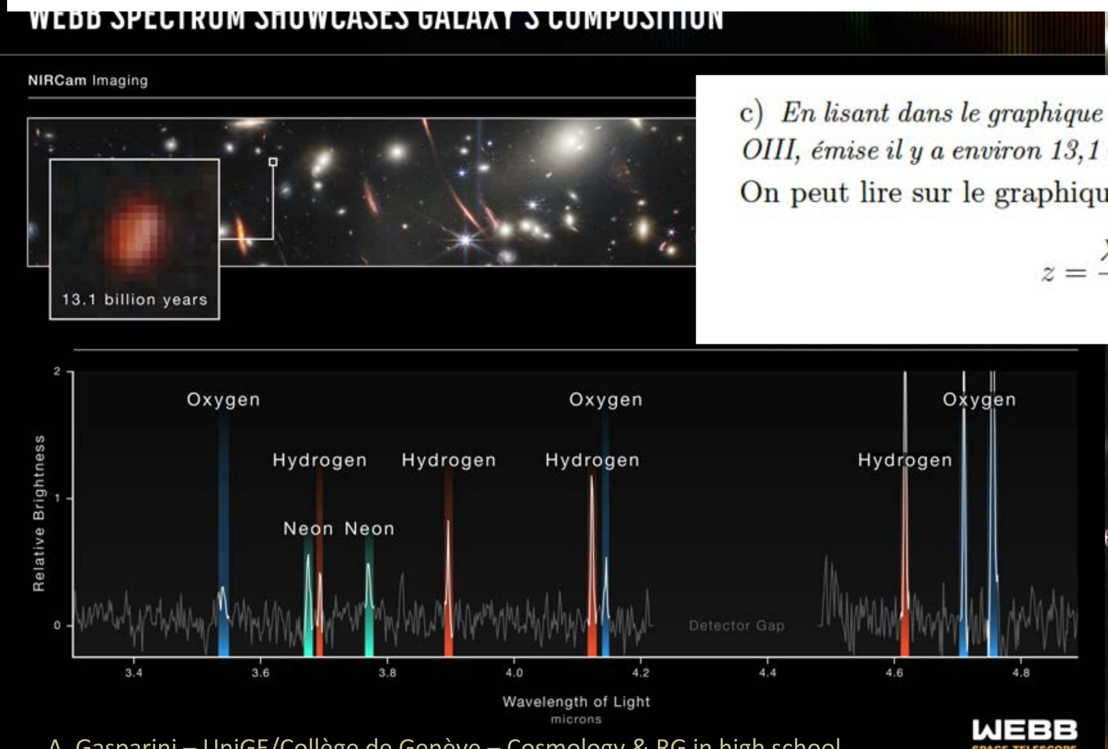
$$\Rightarrow z = \frac{\lambda_0 - \lambda}{\lambda} = \frac{1,2 - 0,486}{0,486} = 1,5$$

c) *Est-ce que cette source se trouve à l'intérieur du rayon de Hubble ? Pourquoi ? Expliquer sa réponse par les formules.*

Non, car le rayon de Hubble est le rayon délimitant la zone autour de nous avec $z < 1 \iff v > c$, c'est à dire tout point à distance d tel que

$$z = \frac{v}{c} < 1 \iff v = H_0 \cdot d < c \iff d < \frac{c}{H_0} = r_H$$

et le redshift de cette source dépasse 1.



c) *En lisant dans le graphique la valeur de la longueur d'onde observée pour la ligne de l'oxygène OIII, émise il y a environ 13,1 milliards d'années par cette source lointaine, calculer son redshift.*

On peut lire sur le graphique $\lambda_0 = 4,75\,\mu$m et on sait que $\lambda = 0,5007\,\mu$m, donc

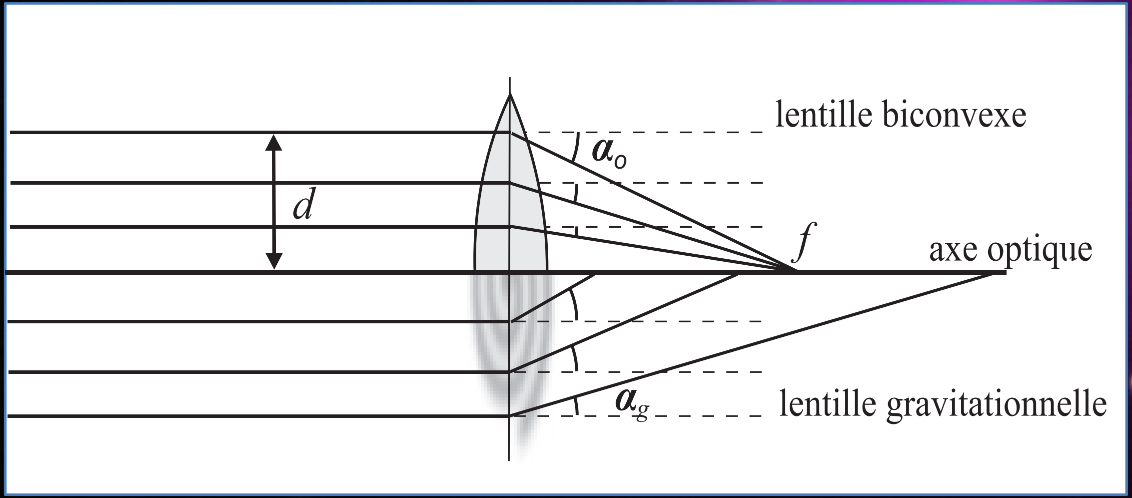
$$z = \frac{\lambda_0 - \lambda}{\lambda} = \frac{4,75 - 0,5007}{0,5007} = 8,49 \approx 9\,.$$

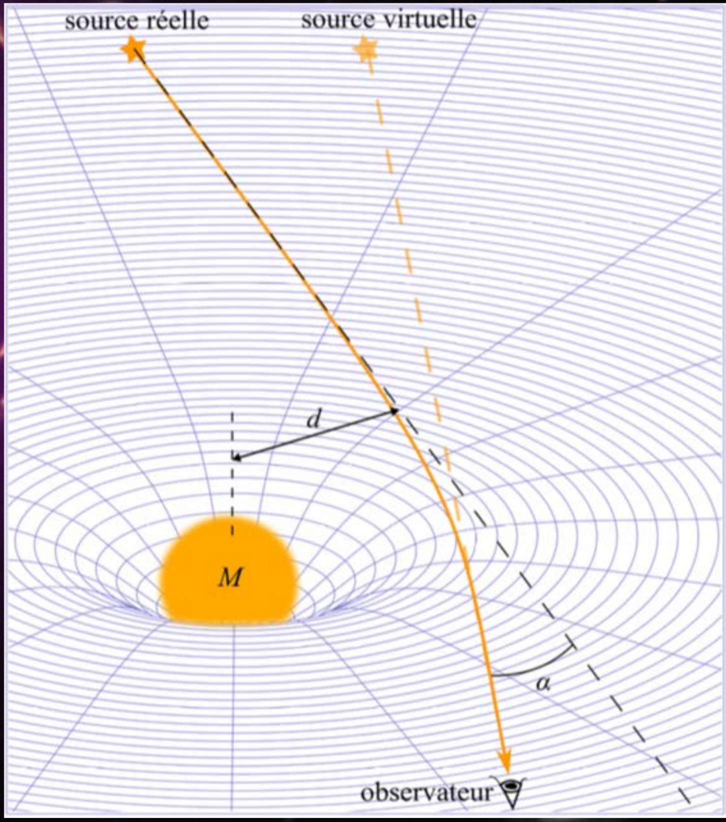
Examples of activities

Gravitational Lensing

- Deflection angle
  - Dimension analysis
  - Parallel geodetics behaviour

$$\alpha \propto \frac{GM}{c^2 d}$$



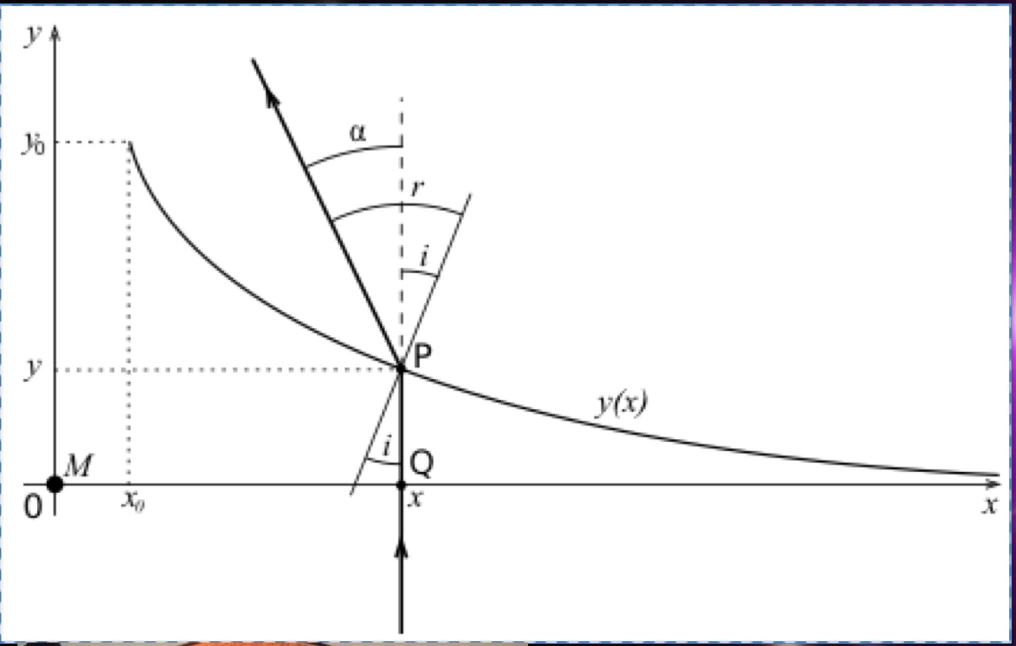


  - Newtonian derivation (factor ½)

- What shape of optical lens to obtain this 1/d behaviour?

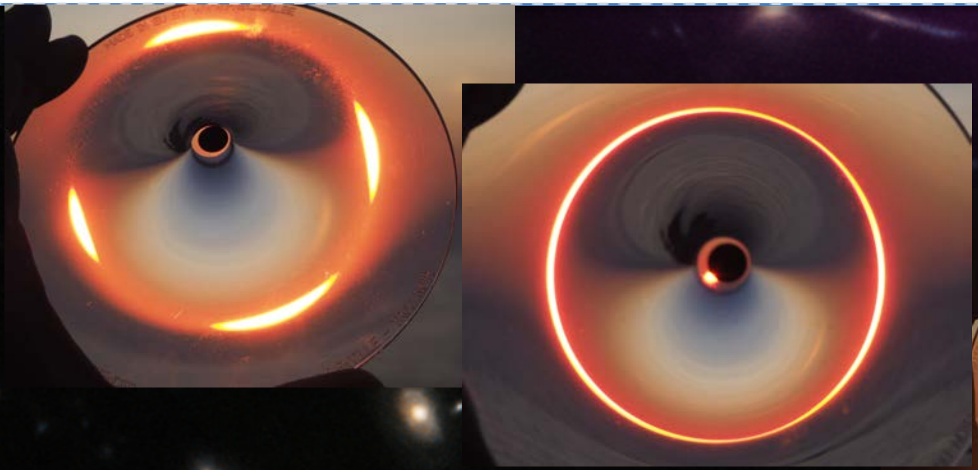
Gravitational Lensing

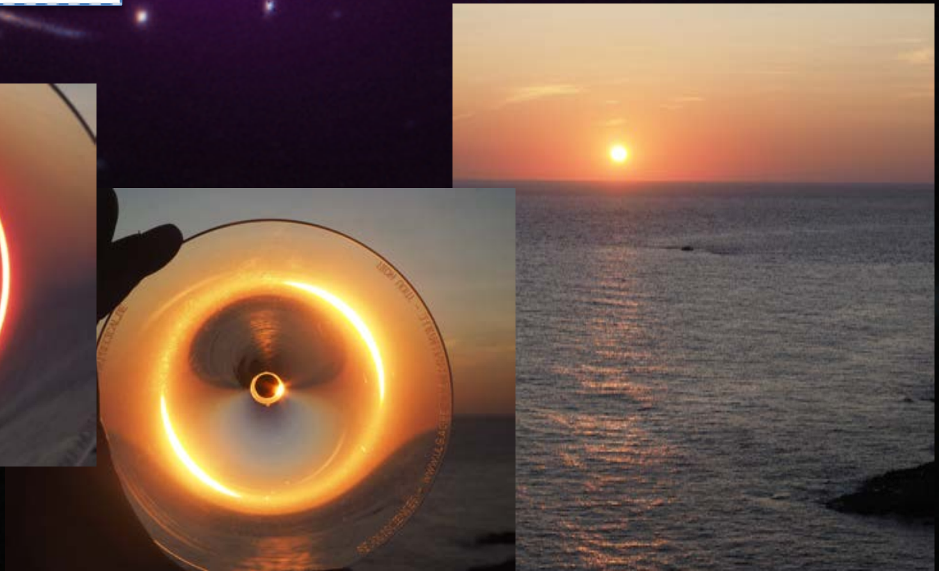
- Integrate to find the corresponding optical lens profile



$$\frac{dy}{dx} = -i(x) = -(\alpha - r) \propto -\frac{1}{x}$$

$$\Rightarrow \ y(x) \propto -\int_{x_0}^{x} \frac{1}{x}\, dx = \ln\left(\frac{1}{x}\right)$$

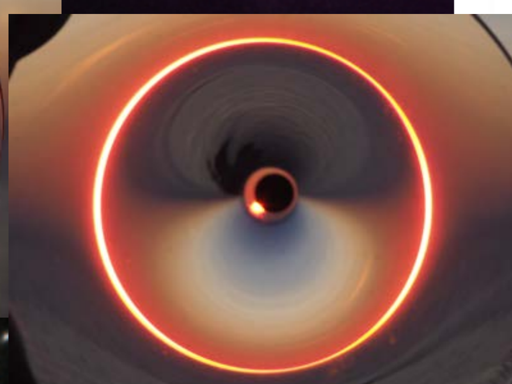


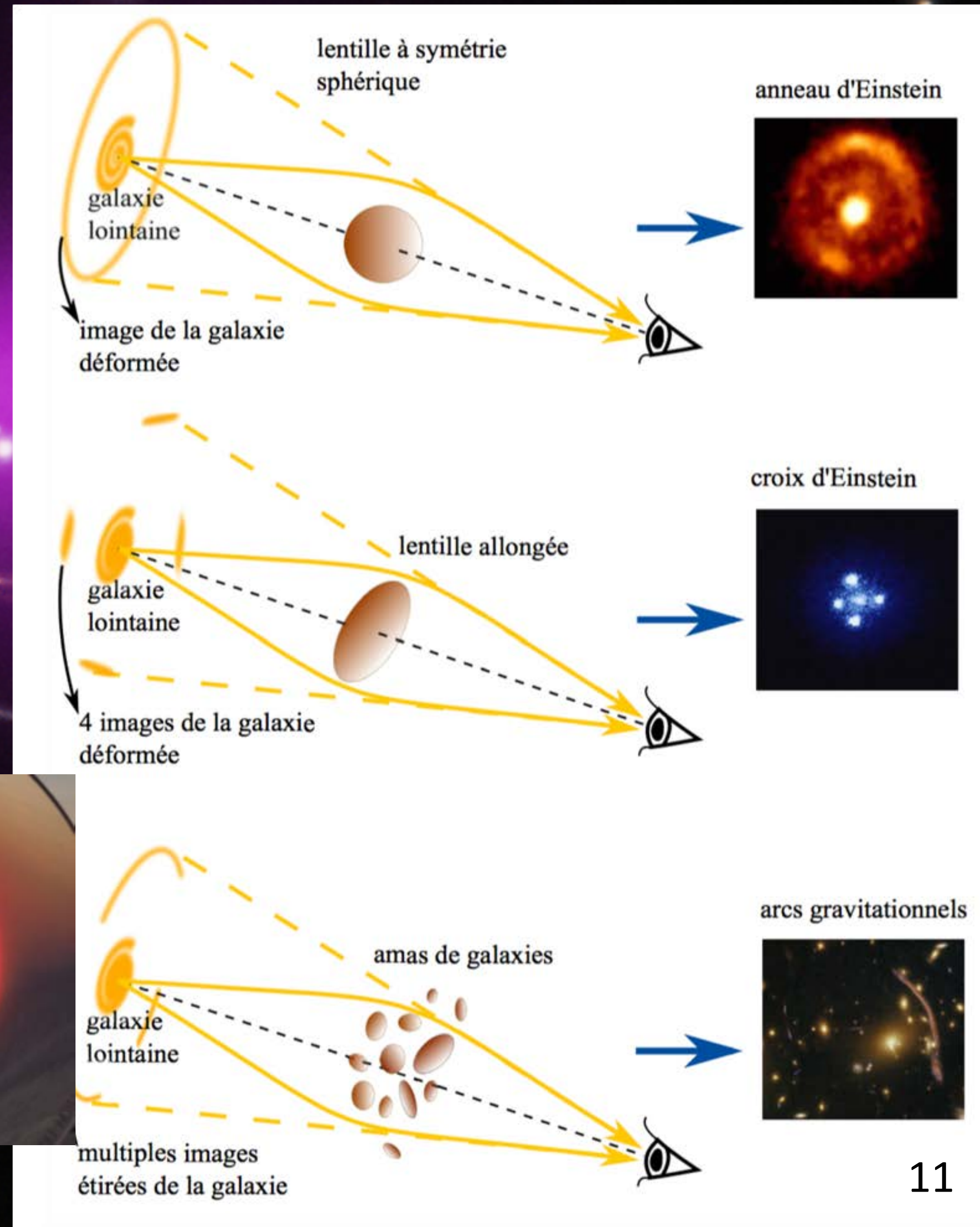


Gravitational Lensing

- Trigonometry & symmetry to find θ_{Einst} as a function of M_{lens}

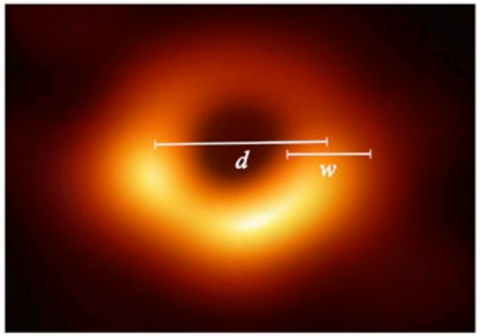
$$\theta_{Einst} = \sqrt{\frac{4GMD_{SL}}{c^2 D_{SO} D_{LO}}}$$





Black holes

b) Expliquer pourquoi l'image du disque d'accrétion est un anneau, même si son axe de rotation ne pointe pas vers l'observateur. De quel phénomène s'agit-il ? (Faire un schéma si nécessaire.)



Crédit : Jifeng, L. *et al.*, Nature, Vol. 575, 618–621 (2019).

Pour les calculs qui suivent, utiliser les données ci-dessous :

Masse du trou noir : $M = 6,5 \cdot 10^9 M_\odot$

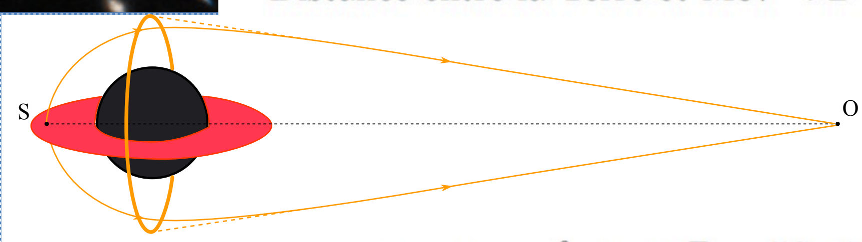
Distance entre la Terre et M87* : $D = 16,8$ Mpc

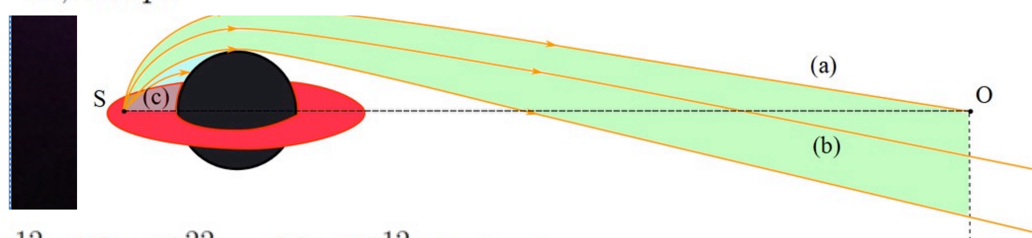
The Astrophysical Journal Letters, 875:L1 (17pp), 2019 April 10

Table 1
Parameters of M87*

Parameter	Estimate
Ring diameter [a] d	42 ± 3 μas
Ring width [a]	<20 μas
Crescent contrast [b]	$>10:1$
Axial ratio [a]	$<4:3$
Orientation PA	150°–200° east of north
$\theta_g = GM/Dc^2$ [c]	3.8 ± 0.4 μas
$\alpha = d/\theta_g$ [d]	$11^{+0.5}_{-0.3}$
M [c]	$(6.5 \pm 0.7) \times 10^9 M_\odot$
Parameter	**Prior Estimate**
D [e]	(16.8 ± 0.8) Mpc
M(stars) [e]	$6.2^{+1.1}_{-0.6} \times 10^9 M_\odot$
M(gas) [e]	$3.5^{+0.9}_{-0.3} \times 10^9 M_\odot$

Notes.
[a] Derived from the image domain.
[b] Derived from crescent model fitting.
[c] The mass and systematic errors are averages of the three methods (geometric models, GRMHD models, and image domain ring extraction).
[d] The exact value depends on the method used to extract d, which is reflected in the range given.
[e] Rederived from likelihood distributions (Paper VI).





$$r_{\text{ombre}} = \theta_{\text{ombre}} \cdot D = 53 \cdot 10^{-12} \cdot 52 \cdot 10^{22} = 28 \cdot 10^{12}\ \text{m}\ > r_S$$

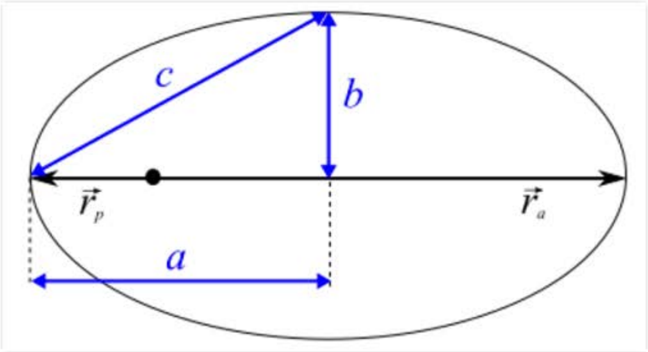
$$\Rightarrow \frac{r_{\text{ombre}}}{r_S} = \frac{28 \cdot 10^{12}\ \text{m}}{19 \cdot 10^{12}\ \text{m}} = 1,5\ .$$

Examples of activities

Black holes

Find the mass of SgrA* using Kepler laws for the orbit of S2

Find its r_S and compare with S14 aphelion ⇨ conclude that Sgr A* is a BH



a) *À partir des données de l'orbite de S2, déterminer la masse de Sgr A*. Exprimer le résultat en kg et en masses solaires.*

$T = 16,05$ années $= 5,065 \cdot 10^8$ s. Le demi grand axe a de l'orbite elliptique vaut

$$a = r_P + c = r_P + ae \quad \Rightarrow \quad a(1-e) = r_P$$

$$\Rightarrow \quad a = \frac{r_P}{1-e} = \frac{1,761 \cdot 10^{13}}{1-0,885} = 1,531 \cdot 10^{14} \text{ m}\,.$$

En utilisant la troisième loi de Kepler on peut trouver la masse du trou noir :

$$\frac{a^3}{T^2} = \frac{GM}{4\pi^2} \quad \Rightarrow \quad M = \frac{4\pi^2 a^3}{T^2 G} = \frac{4\pi^2 \cdot 1,531^3 \cdot 10^{42}}{5,065^2 \cdot 10^{16} \cdot 6,67 \cdot 10^{-11}} = 8,28 \cdot 10^{36} \text{ kg}$$

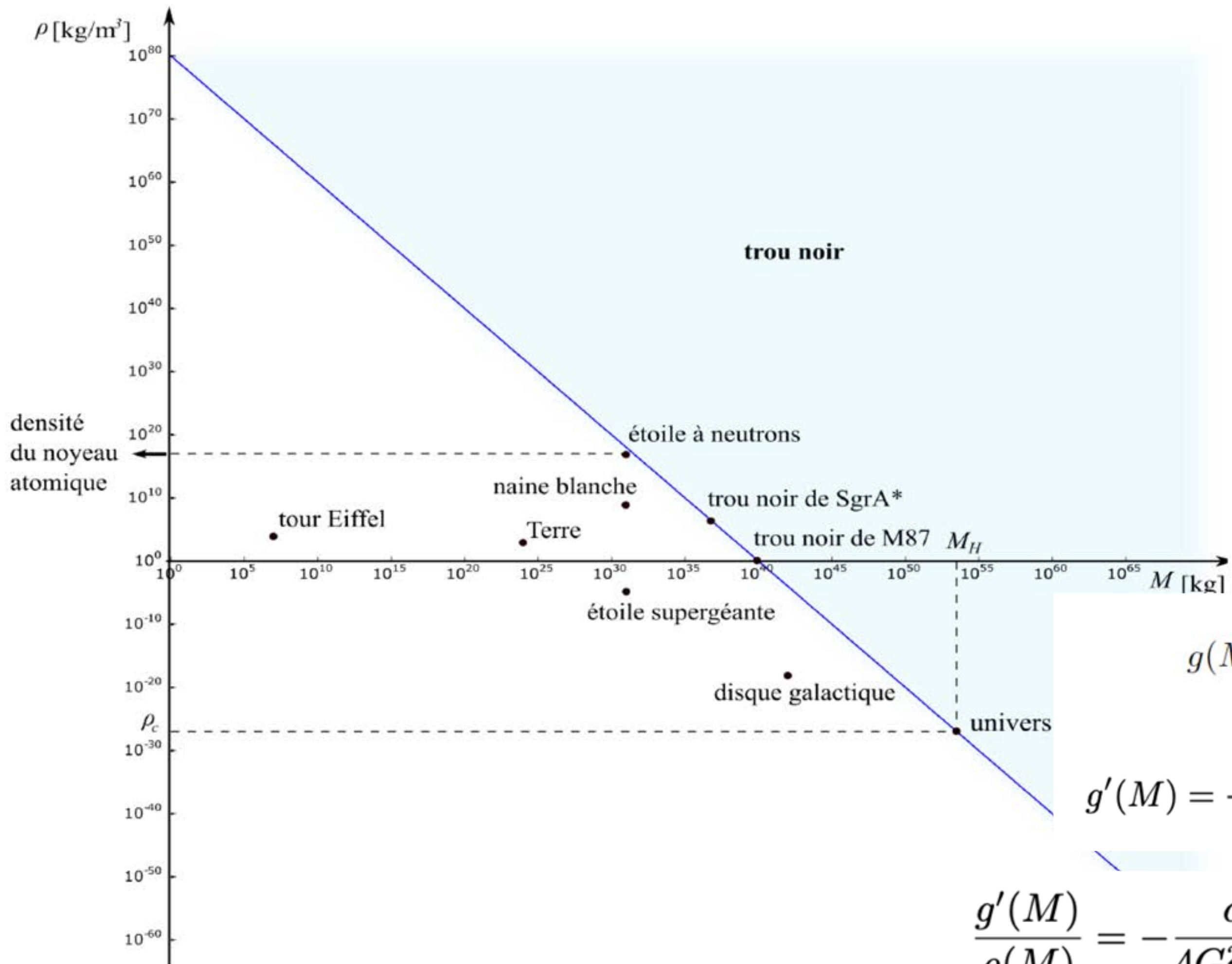
$$\Rightarrow \quad M = 8,28 \cdot 10^{36} \text{ kg} = 4,16 \cdot 10^6 \, M_\odot\,.$$

Black holes

e) *Calculer la densité moyenne de Sgr A* et la comparer avec celle d'un trou noir stellaire.*

On peut trouver ρ à partir de la formule dérivée au cours ou en calculant le rapport entre la masse et le volume du trou noir :

$$\rho = \frac{M}{V} = \frac{M}{4/3\pi\, r_S^3} = \frac{8,28 \cdot 10^{36}}{4/3\pi \cdot 12,3^3 \cdot 10^{27}} = 1,06 \cdot 10^6 \ \mathrm{kg/m^3}\,.$$



- Calculate the average ρ of Sgr A*
- Find the general formula of the average ρ_{BH} decreasing as M^{-2}
- Tide effect and density of a BH

$$g(M) = \frac{c^2}{2r_S} = \frac{c^2}{2 \cdot 2GM/c^2} = \frac{c^4}{4GM}$$

$$g'(M) = -\frac{2GM}{r_S^3} = -\frac{2GM}{(2GM/c^2)^3} = -\frac{c^6}{4G^2M^2}\,,$$

$$\frac{g'(M)}{\rho(M)} = -\frac{c^6}{4G^2M^2} \Big/ \frac{3c^6}{32\pi G^3 M^2} = -\frac{8\pi G}{3}$$

Tide effect near the horizon

a) *Estimer la force de marée axiale créée par Gargantua sur la planète Miller.*

$$r = 10 \cdot r_S = 2,949 \cdot 10^{12}\ \text{m} \quad ; \quad d = 2\,R_T = 2 \cdot 6,378 \cdot 10^6\ \text{m} = 1,2756 \cdot 10^7\ \text{m}$$

$$F_{\text{marée}} = \frac{m}{2} \cdot |g'(r)| \cdot d = \frac{m}{2} \cdot \frac{2GM}{r^3} \cdot d = \frac{GM \cdot m \cdot d}{r^3}$$

$$= \frac{6,673 \cdot 10^{-11} \cdot 1,989 \cdot 10^{38} \cdot 5,972 \cdot 10^{24} \cdot 1,2756 \cdot 10^7}{(2,949 \cdot 10^{12})^3}$$

$$= 3,94 \cdot 10^{-11+38+24+7-36} = 3,94 \cdot 10^{22}\ \text{N}\ .$$

b) *Comparer le résultat du point (a) à la force de marée de la Lune sur la Terre (app. B.3).*

$$\frac{F_{\text{marée Gargantua/Miller}}}{F_{\text{marée Lune/Terre}}} = \frac{3,94 \cdot 10^{22}\ \text{N}}{6,5 \cdot 10^{18}\ \text{N}} = 6,1 \cdot 10^3$$

c) *Quelle serait la force de marée axiale sur Miller si elle était sur l'horizon de Gargantua ?*

Si $r \cong r_S$ on peut refaire le calcul ci-dessus avec un r dix fois plus petit, ce qui donnera un résultat $10^3 = 1\,000$ fois plus grand. On peut également utiliser la formule dérivée au paragraphe 5.5 du cours :

$$F_{\text{marée}} = \frac{c^6 m d}{8 G^2 M^2} = \frac{(3 \cdot 10^8)^6 \cdot 5,972 \cdot 10^{24} \cdot 1,2756 \cdot 10^7}{8 \cdot (6,673 \cdot 10^{-11} \cdot 1,989 \cdot 10^{38})^2}$$

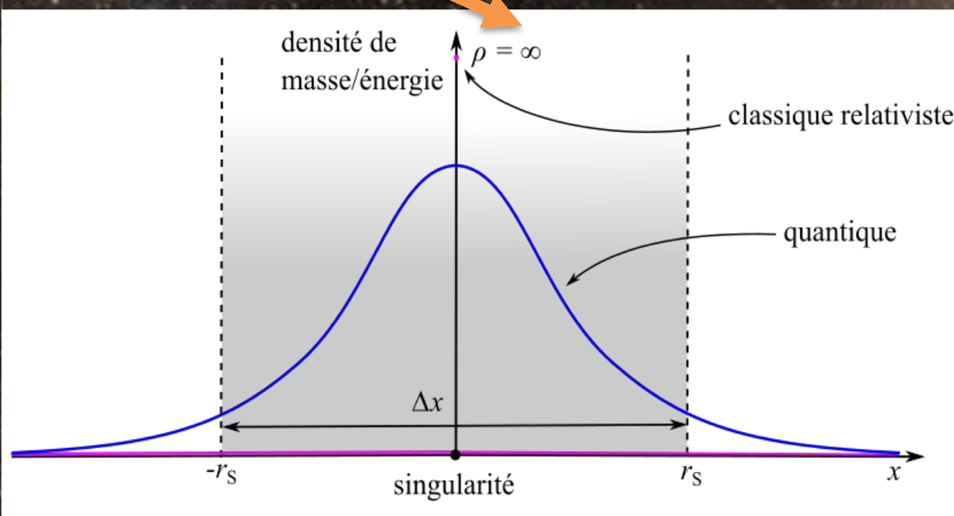
$$= 3,94 \cdot 10^{48+24+7-2(38-11)} = 3,94 \cdot 10^{25}\ \text{N}\ .$$

Temperature and evaporation

$$T \propto \frac{1}{M}$$

Microscopic interpretation with QM

- Pair production in a vacuum (Hawking interpretation)
- Tunneling



Estimate the temperature of a BH with the indetermination principle

$$\Delta E = \Delta p \cdot c \approx \frac{\hbar c}{2\,\Delta x} \lessapprox \frac{\hbar c}{4\,r_S}\,.$$

$$\Delta E \approx \frac{\hbar c^3}{8\,GM}$$

$$T \approx \frac{\Delta E}{k_B} \approx \frac{\hbar c^3}{8\,k_B GM}\,. \qquad (5.37)$$

Ce résultat correspond à un facteur π près à la formule de Hawking (5.23) vue auparavant.

Examples of activities

Energy-mass conservation + Stefan-Boltzmann law (+ some rough but well justified approximations)
➪ estimate the BH evaporation time t_e

$$P_L = \frac{dE}{dt} = -\frac{dMc^2}{dt} = 4\pi r_S^2 \cdot \sigma \cdot T^4. \qquad (5.29)$$

En remplaçant les équations du rayon de Schwarzschild $r_S(M) = 2GM/c^2$ et de la température du trou noir (5.23) en fonction de la masse dans l'équation (5.29), nous obtenons

$$-\frac{dMc^2}{dt} = 4\pi \left(\frac{2GM}{c^2}\right)^2 \cdot \sigma \cdot \left(\frac{\hbar c^3}{8\pi k_B GM}\right)^4$$
$$\Rightarrow \quad \frac{dMc^2}{dt} = -\frac{\sigma \cdot \hbar^4 c^8}{256\pi^3 k_B^4 (GM)^2}\,. \qquad (5.30)$$

Nous remplaçons σ par son expression (5.28) et écrivons l'équation (5.30) comme

$$\frac{dMc^2}{dt} = -\frac{\frac{\pi^2 k_B^4}{60\hbar^3 c^2} \cdot \hbar^4 c^8}{256\pi^3 k_B^4 (GM)^2} \quad \Rightarrow \quad \frac{dMc^2}{dt} = -\frac{\hbar c^6}{15360\pi G^2 M^2}$$
$$\Rightarrow \quad \frac{dM}{dt} = -\frac{\hbar c^4}{15360\pi G^2 M^2} \quad \Rightarrow \quad M^2 dM = -\frac{\hbar c^4}{15360\pi G^2}\,dt\,. \qquad (5.31)$$

$$\int_{M(t=0)=M_{\text{in}}}^{M(t=t_e)=0} M^2 dM = -\int_{t=0}^{t=t_e} \frac{\hbar c^4}{15360\pi G^2}\,dt$$
$$\Rightarrow \quad 0 - \frac{M_{\text{in}}^3}{3} = -\frac{\hbar c^4}{15360\pi G^2}(t_e - 0)$$
$$\Rightarrow \quad -t_e = -\frac{15360\pi G^2}{\hbar c^4} \cdot \frac{M_{\text{in}}^3}{3}$$
$$\Rightarrow \quad t_e = \frac{5120\pi G^2 M_{\text{in}}^3}{\hbar c^4}\,. \qquad (5.32)$$

Examples of activities

$$t_e \approx \frac{5 \cdot 10^3 \cdot 3 \cdot (7 \cdot 10^{-11}\ \mathrm{m}^3/(\mathrm{s}^2\ \mathrm{kg}))^2 {M_{\mathrm{in}}}^3}{1 \cdot 10^{-34}\ \mathrm{kg}/(\mathrm{m/s})^2 \cdot \mathrm{s} \cdot (3 \cdot 10^8\ \mathrm{m/s})^4}$$

$$\approx 9 \cdot 10^{-17}\mathrm{s} \cdot (M_{\mathrm{in}}/\,\mathrm{kg})^3 \qquad (5.33)$$

$$\boxed{\Rightarrow \quad t_e \sim 10^{-16} M_{\mathrm{in}}^3 \frac{\mathrm{s}}{\mathrm{kg}^3}} \qquad (5.34)$$

b) *Déterminer l'ordre de grandeur du temps d'évaporation d'un trou noir de (1) $M_{\mathrm{in}} = 1$ kg, (2) $M_{\mathrm{in}} = M_{\mathrm{Terre}}$ et (3) $M_{\mathrm{in}} = 10\ M_{\odot}$.*

1. $t_e \sim 10^{-16}\ \frac{\mathrm{s}}{\mathrm{kg}^3} \cdot M_{\mathrm{in}}^3 = 10^{-16}\ \frac{\mathrm{s}}{\mathrm{kg}^3} \cdot (1\ \mathrm{kg})^3 \sim 10^{-16}\ \mathrm{s}$;

2. $t_e \sim 10^{-16}\ \frac{\mathrm{s}}{\mathrm{kg}^3} \cdot (6 \cdot 10^{24}\ \mathrm{kg})^3 \approx 2 \cdot 10^{58}\ \mathrm{s} \approx 7 \cdot 10^{50}$ années $\sim 10^{51}$ années ;

3. $t_e \sim 10^{-16}\ \frac{\mathrm{s}}{\mathrm{kg}^3} \cdot (2 \cdot 10^{31}\ \mathrm{kg})^3 \approx 8 \cdot 10^{77}\ \mathrm{s} \approx 3 \cdot 10^{70}$ années $\sim 10^{70}$ années .

c) *Quelle devrait être la masse initiale d'un trou noir pour qu'il s'évapore en une durée de l'ordre de l'âge de l'univers, $t_{\mathrm{evap}} \sim 10^{10}$ années ?*

$$t_e \approx 9 \cdot 10^{-17}\ M_{\mathrm{in}}^3 = 1,4 \cdot 10^{10}\ \text{années} = 4,4 \cdot 10^{17}\ \mathrm{s}$$

$$\Rightarrow \quad M_{\mathrm{in}}^3 \approx \frac{4,4 \cdot 10^{17}\ \mathrm{s}}{9 \cdot 10^{-17}\ \mathrm{s/kg}^3} = 5 \cdot 10^{33}\ \mathrm{kg}^3$$

$$\Rightarrow \quad M_{\mathrm{in}} \approx \sqrt[3]{5 \cdot 10^{33}\ \mathrm{kg}^3} = 1,7 \cdot 10^{11}\ \mathrm{kg} \approx 2 \cdot 10^{11}\ \mathrm{kg}\ .$$

Examples of activities

Time dilation

Equivalence between
- standard point-mass gravity
- uniform circular motion (artificial gravity)

Equivalence between time dilation
- for uniform circular motion (RR)
- in presence of a spherical mass (GR)

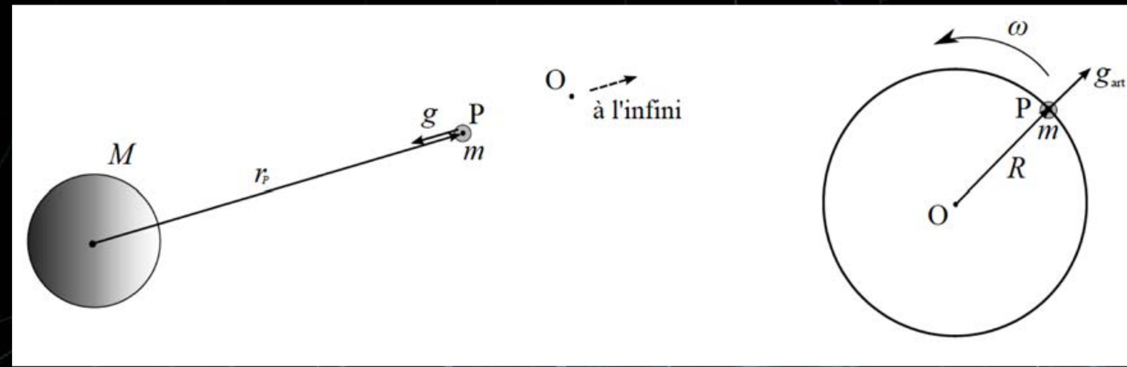


Tableau de comparaison entre gravité et gravité artificielle

Grandeur	Gravité	Gravité artificielle
Accélération de la gravité	$g = -\frac{G\,M}{r^2}$	$g_{\text{art}} = +\omega^2 R$
Force de la pesanteur	$F_g = -\frac{G\,M\,m}{r^2}$	$F_{g\,\text{art}} = +m\,\omega^2\,R$
Energie potentielle gravitationnelle au point P	$E_{g\,\text{P}} = -\frac{GMm}{r_\text{P}}$	$E_{g\,\text{art}\,\text{P}} = -\frac{1}{2}\,m\omega^2 R^2$
Point O, où l'énergie potentielle vaut zéro	à l'infini	au centre du MCU

$$\Delta t = \Delta t_\text{O} = \frac{\Delta t_\text{P}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{\Delta t_\text{P}}{\sqrt{1-\frac{\omega^2 R^2}{c^2}}}$$

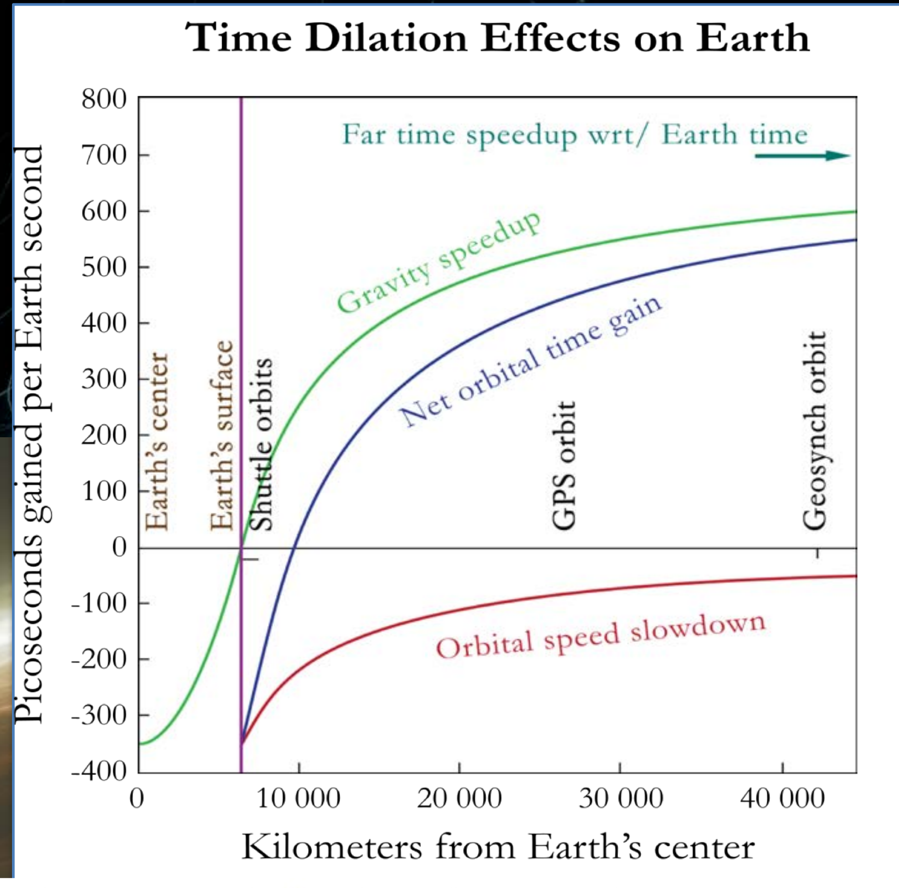
$$\Delta t_\text{P} = \Delta t \cdot \sqrt{1 + \frac{2E_{g\,\text{art}\,\text{P}}}{m\,c^2}}$$

$$\Delta t_\text{P} = \Delta t \cdot \sqrt{1 + \frac{2E_{g\text{P}}}{m\,c^2}} = \Delta t \cdot \sqrt{1 - \frac{2GM}{c^2 r_\text{P}}}$$

Examples of activities

Time dilation

$$\Delta t_g = \Delta t \cdot \sqrt{1 - \frac{2GM}{c^2 r}}$$



Le temps mesuré par Cooper lorsqu'il reste à proximité de l'horizon du trou noir est toujours le temps *propre* $\Delta t_{0\,g} = 2\text{h } 45' = 2{,}75$ h. Pour un observateur au repos sur Terre, où l'effet de la gravité est négligeable, la durée correspondante est (effet de la relativité générale)

$$\Delta t_g = \frac{1}{\sqrt{1 - 2GM/(c^2 r)}} \cdot \Delta t_{0\,g}$$

$$\frac{2GM}{c^2 r} = \frac{2 \cdot 6,67 \cdot 10^{-11} \cdot 1,99 \cdot 10^{38}}{9 \cdot 10^{16} \cdot 3 \cdot 10^{11}} = 0,9832$$

$$\Rightarrow \quad \Delta t_g = \frac{1}{\sqrt{1 - 0,9832}} \cdot 2,75 = 7,717 \cdot 2,75 = 21,22 \text{ h}\,.$$

Cosmological equations

- Friedmann equation form energy-mass conservation
- Find the cosmological constant for a static universe
- Show that it leads to an unstable solution

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\frac{\rho_{m0}}{a^3} + \Lambda_E \quad \Rightarrow \quad \dot{a}^2 = \frac{8\pi G}{3}\frac{\rho_{m0}}{a} + \Lambda_E\, a^2\ ;$$

on impose $\dot{a}_0 = \dot{a}(t_0) = 0$ (condition d'univers statique) et on trouve

$$\dot{a}_0^2 = \frac{8\pi G \rho_{m0}}{3} + \Lambda_E = 0 \quad \Rightarrow \quad \Lambda_E = -\frac{8\pi G \rho_{m0}}{3}\ .$$

Si on prend comme valeur de la densité $\rho_{m0} \approx 1 \cdot 10^{-27}\ \mathrm{kg/m^3}$ on obtient

$$\Lambda_E \approx -6 \cdot 10^{-37} \sim\ -10^{-36}\ \mathrm{s}^{-2}\ < 0\ .$$

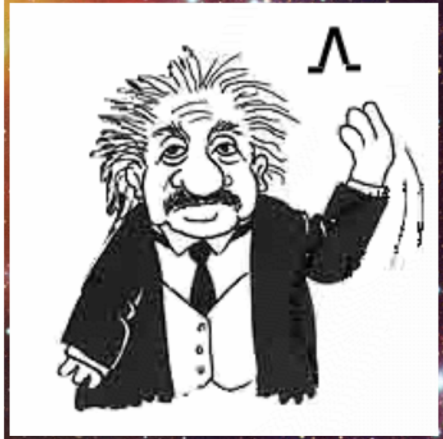
c) *Montrer que, même si aujourd'hui l'univers était statique ($\dot{a}(t_0) = 0$), il serait instable : une valeur de la vitesse infinitesimale ($\dot{a}(t_0) = \epsilon \ll 1$) impliquerait une accélération $\ddot{a}(t_0)$ non nulle. Qu'en déduire ?*

$$\dot{a}^2 = \frac{8\pi G}{3}\frac{\rho_{m0}}{a} + \Lambda_E a^2 \quad \Rightarrow \quad 2\dot{a}\cdot\ddot{a} = \frac{8\pi G}{3}\cdot\left(-\frac{\rho_{m0}}{a^2}\cdot\dot{a}\right) + 2\Lambda_E a\dot{a}\ ;$$

en simplifiant par $\dot{a} = \epsilon$ on a :

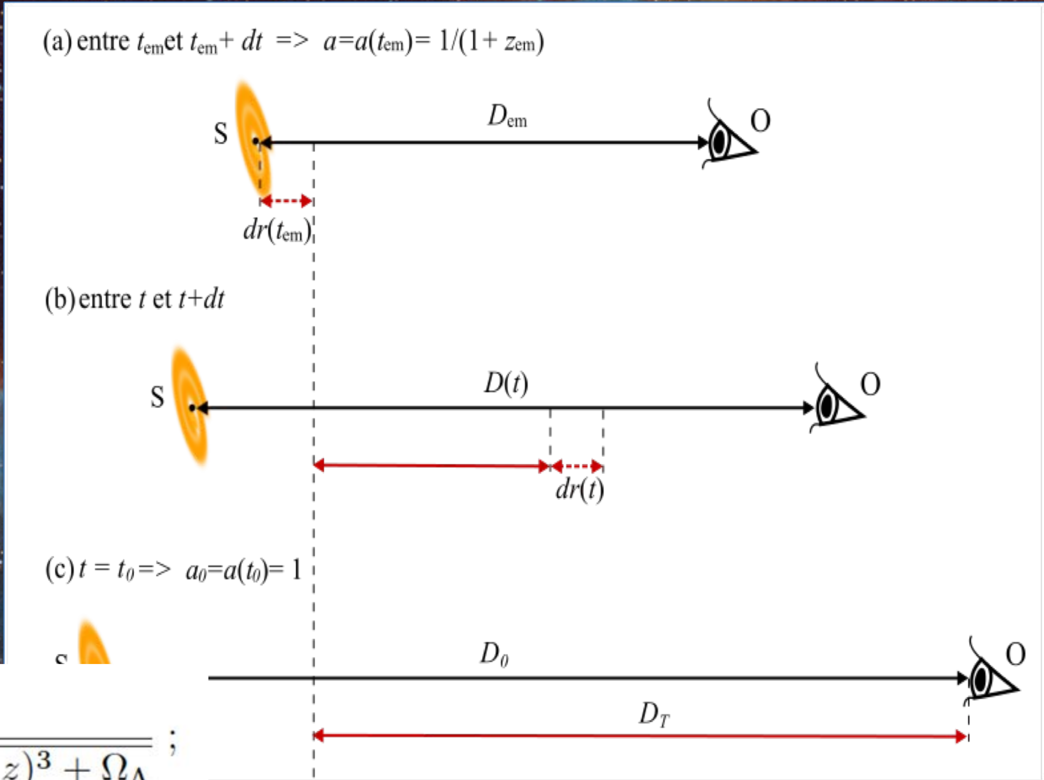
$$\ddot{a} = -\frac{8\pi G}{6}\cdot\frac{\rho_{m0}}{a^2} + \Lambda_E a \quad \Rightarrow \quad \ddot{a}(t_0) = -\frac{8\pi G \rho_{m0}}{6} + \Lambda_E = -\frac{8\pi G \rho_{m0}}{6} - \frac{8\pi G \rho_{m0}}{3}$$

$$\Rightarrow \quad \ddot{a} = -4\pi G \rho_{m0} < 0\ .$$



Cosmological distances

Finite value of c + expansion
⇨ on cosmological scales the common notion of distance splits into 5 different quantities



$$D_T(z_s) = \int_{t_{em}}^{t_0} dr = \frac{c}{H_0}\int_0^{z_s} \frac{dz}{(1+z)\cdot\sqrt{\Omega_m\cdot(1+z)^3+\Omega_\Lambda}} \;;$$

$$D_0(z_s) = \int_{t_{em}}^{t_0} dr_0 = \frac{c}{H_0}\int_0^{z_s} \frac{dz}{\sqrt{\Omega_m\cdot(1+z)^3+\Omega_\Lambda}} \;;$$

$$D_A(z_s) = D_{em}(z_s) = a_{em}\cdot D_0$$

$$= \frac{c}{H_0(1+z_s)}\int_0^{z_s} \frac{dz}{\sqrt{\Omega_m\cdot(1+z)^3+\Omega_\Lambda}} \;;$$

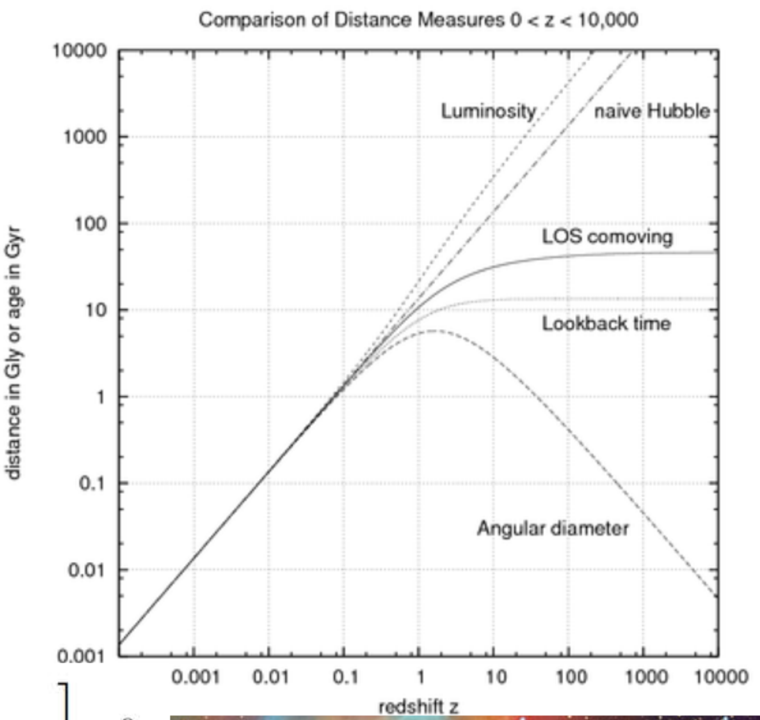
$$D_L(z_s) = \frac{1}{a_{em}^2}\cdot D_{em} = \frac{1}{a_{em}}\cdot D_0$$

... with different behavior as a function of the redshift z

Examples of activities

Cosmological distances

- study the limit cases
- find the luminosity distance (D_L) as a function of Ω_Λ and Ω_m , and compare with the Supernova Cosmology Project data



$$\frac{dD_{\rm em}(z_s)}{dz_s} = \frac{2c}{H_0}\cdot\left[-\frac{1}{(1+z_s)^2}\cdot\left(1-\frac{1}{(1+z_s)^{1/2}}\right)+\frac{1}{(1+z_s)}\cdot\frac{1}{2}\cdot\frac{1}{(1+z_s)^{3/2}}\right]=0$$

$$\Rightarrow\quad -\frac{1}{(1+z_s)^2}\cdot\left(1-\frac{1}{(1+z_s)^{1/2}}\right)+\frac{1}{2}\cdot\frac{1}{(1+z_s)^2}\cdot\frac{1}{(1+z_s)^{1/2}}=0$$

$$\Rightarrow\quad -\frac{1}{(1+z_s)^2}\cdot\left(-1+\frac{1}{(1+z_s)^{1/2}}+\frac{1}{2\cdot(1+z_s)^{1/2}}\right)=0$$

$$\Rightarrow\quad -1+\frac{3}{2\cdot(1+z_s)^{1/2}}=0 \quad\Rightarrow\quad (1+z_s)^{1/2}=\frac{3}{2}$$

$$\Rightarrow\quad 1+z_s=\frac{9}{4}\qquad\Rightarrow\qquad z_s=\frac{5}{4}=1,25\,.$$

- find the value of z that maximizes the proper distance at the time of emission (D_{em})

(with $\Omega_\Lambda = 0$ and $\Omega_m = 1$)

More exemples of activities

- CMB power spectrum and curvature of the universe (ch. 2)
- Equivalence principle, inertia and gravitation for Einstein vs. Newton (ch. 3)
- Primordial nucleosynthesis and abundance of light elements (ch. 7)
- Fusion temperature and tunneling (ch. 7)
- Gravitational radiation (ch. 8)
  - Comparing quadrupole (GW) vs. dipole (EMW) emission
  - Why only astrophysical masses can produce such waves ?
  - Why relativistic sources are needed for detection ? (BH or NS)
  - Why are these waves so important in the nowadays physics ?
  - Relation M_{source} / frequency / detector's size
  - + all the exercises you can do with waves (traditional curriculum)

Study features

- **Sample**: last year high-school students choosing the PHY complementary option (CO) course in Geneva
  ⇨ **not** physics specialized maturity (no PHY specific option)
- 4 class-groups form 2020 to 2024, with global ratio gender of 1.3

School year	N	N_F	N_F / N_M complementary option CO)	N_F / N_M specialized physics courses (SO)	N_F / N_M in the high school
2020-21	16	8	1.0	0.28	1.3
2021-22	22	12	1.2	0.30	1.3
2022-23	20	12	1.5	0.33	1.2
2023-24	12	6	1.0	0.27	1.2
Total	70	38	$\boxed{1.2}$	$\boxed{0.30}$	$\boxed{1.3}$

- **Duration**: one semester, from August to January (36 periods)
- **Content**: Universe scales, structures, composition, acceleration, basics of GR, gravitational lensing, black holes (ch. 1 to 5)

Ref : A. Gasparini, A. Müller, F. Stern, L. Weiss, *“Cosmology and general relativity in upper secondary school through new targeted teaching materials: A study on student learning and motivation”, Phys. Rev. Phys. Educ*. Res. **21**, 020107 (2025)

Conceptual learning

- Strong impact of the course on *conceptual learning* of cosmology, based on a CMQ of 14 newly created items following on the course content

Learning : PRE/POST comparaison				
Index	P_{pre} (SD)	P_{post} (SD)	Cohen's *d*	Gain = $(\langle P_{post}\rangle - \langle P_{pre}\rangle)/(1- \langle P_{pre}\rangle)$
Mean(SD)	**0.35(.18)**	**0.76(.15)**	**2.8*** **	**63%**
[range of values]	[0.06 -> 0.60]	[0.43 -> 0.93]	[0.44* -> 1.59**]	[39% -> 90%]

Medium (*) to large (**) positive size effect on each item, and very large (***) overall effect (*t*-tests with $p < .001$)

- Moreover, 10 items out of 14 revealed a misconception, overtaken after the intervention

Conceptual learning

Item keywords (translated from French)	Right answer	Percent of right answers PRE → POST	Related misconception	% of answers related to the misconception PRE → POST
We can observe other galaxies …	… moving away more quickly as their distance is greater	37% → 76%	… moving away with the same speed/almost at rest	46% → 24%
We call "dark matter" …	matter which does not interact electromagnetically	31% → 93%	… particles of antimatter	37% → 0%
The "cosmic microwave background" is the radiation …	… emitted in the primordial universe when neutral atoms were formed	25% → 91%	… emitted during the formation of the first galaxies	37% → 6%
By "dark energy" we mean …	… the energy at the origin of the change in volume of empty space	31% → 87%	… the energy of dark matter	41% → 3%
A BH 100 times more massive than the other is	10000 times less dense	6% → 43%	… the same / 100 / 10000 times denser	71% → 25%

- Moreover, 10 items out of 14 revealed a misconception, overtaken after the intervention

Affective outcomes

Small effect (*) of the intervention on *curiosity as a state* and *self-concept* in relation to the course, based on standard test items

Affective outcomes : PRE/POST comparaison			
Dependent variable (number of items in the scale)	$Mean_{pre}$ (SD)	$Mean_{post}$ (SD)	Cohen's *d*
Interest (K=5)	**3.94(.82)**	**4.08(.92)**	**0.17**
Curiosity State (K=5)	**3.93(1.02)**	**4.40(.98)**	**0.46***
Self-Concept (K=5)	**3.76(.99)**	**4.02(1.04)**	**0.26***
Relevance of Science (K=8)	**3.67(.88)**	**3.68(.89)**	**0.01**

↳ Participants' interviews confirm that this can be explained by the adequate structure of the course, tailored to the targeted level

↳ Students' responses indicate how a structured and in-depth treatment of certain subjects may not increase their interest, as they are satisfied with learning at their current level, but they remain willing to learn more, aware that this would require a higher level of mathematics and physics

Students’ quotes

Awareness about the physics courses and relevance of science

- *"I thought Newtonian mechanics was insufficient for calculating, for example, the Schwarzschild radius. I discovered that Newtonian mechanics can actually provide good approximations for more complex results.”*
- *"As for Newton's laws and waves, I can explain them now, and I feel more confident about these topics.”*
- *"It's actually the more philosophical side that is highlighted, thanks to the quantitative data, but also to my personal research.”*
- *“I didn't think the temperature of the universe was that low. Our intuition is distorted by our senses and our daily experience.”*
- *"Understanding that we are nothing compared to the universe, realizing that we are a tiny part of what surrounds us, was very striking to me.”*

Students’ quotes

About curiosity versus interest

- *“Curiosity grew because the subject is truly interesting, and there’s so much we don’t yet know. But interest might decrease a bit because once you know the basics, the course inevitably gets more technical with math and physics. As for me, I like astronomy, but I wouldn’t go further now.”*

About the course

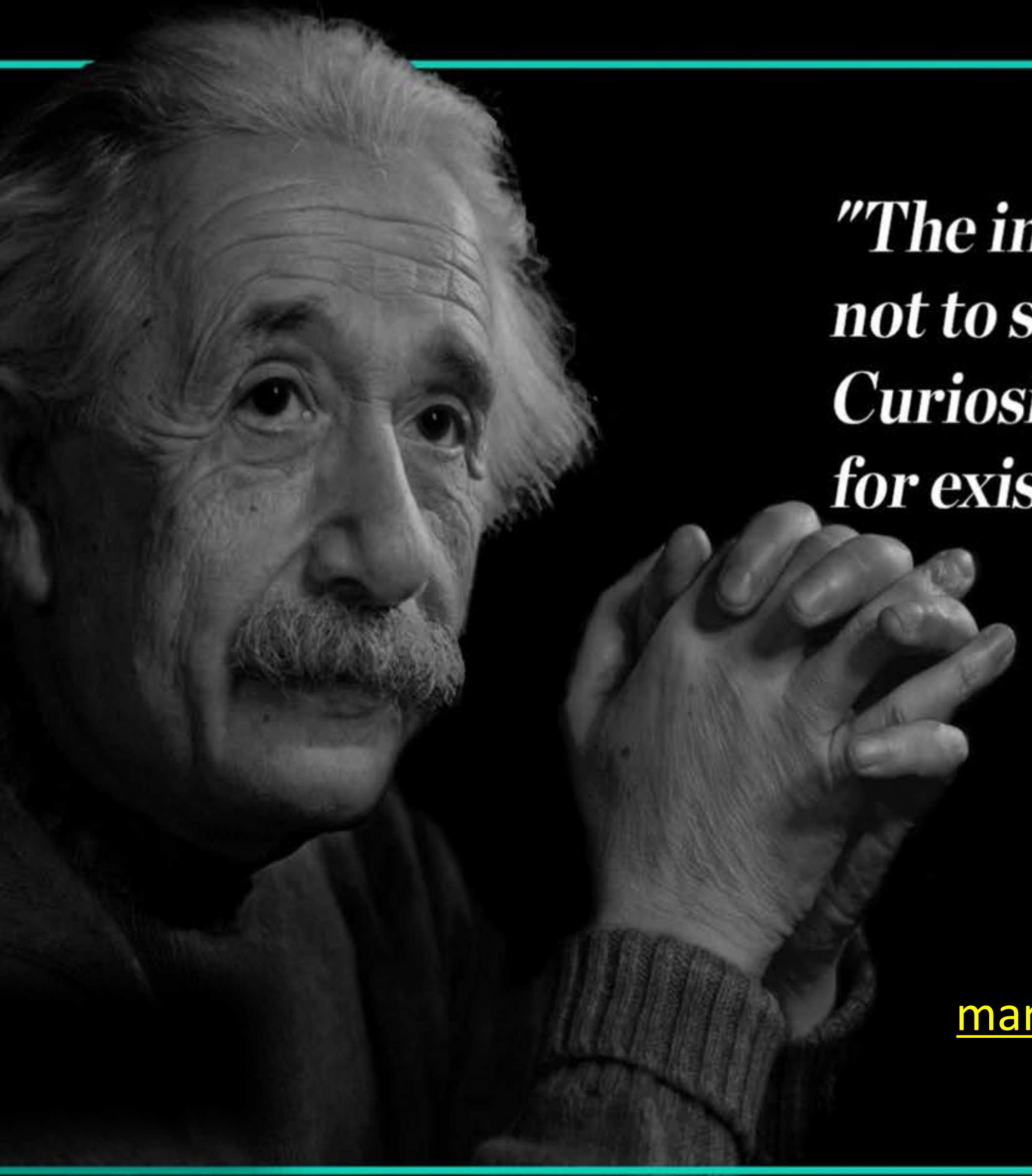
- *“The course came at a very good time because the day after we covered the subject, we saw photos of the black hole in the center of our galaxy in the newspapers and were able to study it.” (about the M87 black hole by the EHT)*
- *“The documents were very precise and very well explained, the book is very well done, the files with summaries helped a lot.”*
- *“It was the most interesting course of the year, I loved it, I suggest that it always be integrated into the physics course.”*

Outlook

- From year 2024-25 the course of C&GR is offered over 2 semesters
  - ↳ Possibility to complete the 8 chapters
  - ↳ Also in Italian (Ticino Canton)

- From 2026 also in English

- A network of collaborating teachers and researchers
  - ↳ improve and expand questioners and interviews to evaluating the impact on students' learning and motivation
  - ↳ update the C&GR educational resources, following the evolution of scientific actualities
  - ↳ create new collaborations for observations (Stellarium Gornergrat, OFXB in Saint Luc)



"The important thing is not to stop questioning. Curiosity has its own reason for existing."

- Albert Einstein

Thank you

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